Exercises Sheet 1

Exercise 1. *1)Let* $X = [0,1)$*. Show that the doubling map* $x \to 2x \pmod 1$ *on* X *is measure-preserving with respect to the Lebesgue measure on* X*.*

2)Let $X = [0,1)$, $g(x) = 2x \pmod 1$, $\mu = Leb$ *and let* $Y = \Pi_{n=1}^{\infty}\{0,1\}$, $h : (x_n)_{n\geq 1} \to (x_{n+1})_{n\geq 1}$, $\nu = (\frac{1}{2}(\delta_0 + \delta_1))^{\otimes \mathbb{N}}$*. Show that they are isomorphic.*

Exercise 2. *Show the cylinder sets* $C_{m,a_0,\cdots,a_n}$ *form a Bool semialgebra* $\mathcal{C}$ *and generate the Borel* σ*-algebra on* $\Lambda^{\mathbb{E}}$*.*

Exercise 3. *Let* X *be compact metric space and* f *a continuous map. A point* x *in* X *is called quasi-periodic if for all neighbor* V *of* x*, there exists an integer sequence* n_k *tends to infinity, such that* $f^{n_k}(x) \in V$ *and* $n_{k+1} - n_k$ *"the return time to* V*" is bounded.*

- *Show any point in a closed minimal set* Y *of* X *is quasi-periodic.*
- *Show* X *contains quasi-periodic points.*
- *Show the inverse: if* x *is quasi-periodic then the orbit closure* $\overline{\{f^n(x), n \in \mathbb{N}\}}$ *is minimal.*

Exercise 4. *Consider symbolic system* $\Lambda^{\mathbb{Z}}$ *and* $\Lambda^{\mathbb{N}}$ *with* $\Lambda = \{0,1\}$*.*

- *Construct a point* ω *in* $\Lambda^{\mathbb{Z}}$ *which is recurrent for* σ *but not for* σ^{-1}*.*
- *Construct a point* ω *which is quasi-periodic but not periodic*
- *Construct a point* ω *which is recurrent but not quasi-periodic.*

Exercise 5. *Construct a part* A *of* $\mathbb{N}$ *such that neither* A *nor* A^c *contains an infinite arithmetic sequence.*